Higher Dimensional Percolation

Paul Duncan

Department of Mathematics
OSU

April 27, 2019

Bond Percolation

- Consider the integer lattice obtained by connected each vertex in $\mathbb{Z}^d \subset \mathbb{R}^3$ to its nearest neighbors. We can construct a random subgraph by starting with $\mathbb{Z}^d$ and adding each possible edge with probability p independently.
- A classical problem is to find the threshold for p, called $p_c(\mathbb{Z}^d)$ at which an infinite connected component appears with positive probability.

Bond Percolation

- Consider the integer lattice obtained by connected each vertex in $\mathbb{Z}^d \subset \mathbb{R}^3$ to its nearest neighbors. We can construct a random subgraph by starting with $\mathbb{Z}^d$ and adding each possible edge with probability p independently.
- A classical problem is to find the threshold for p, called $p_c(\mathbb{Z}^d)$ at which an infinite connected component appears with positive probability.

Plaquette Percolation

- Construct a random complex by starting with the full integer lattice and adding 2-dimensional faces with probability p independently.

Theorem (Aizenman, Chayes, Chayes, Frölich, Russo)

Let γ be a planar rectangular loop in the integer lattice $\mathbb{L} \subset \mathbb{R}^3$, and let W_γ be the event that there is a plaquette surface with γ as its boundary. Then we have

$$\mathbb{P}(W_\gamma) \sim \begin{cases} \exp(-\alpha(p)\mathrm{Area}(\gamma)) & p < 1 - p_c(\mathbb{Z}^3) \\ \exp(-\beta(p)\mathrm{Per}(\gamma)) & p > 1 - p_c(\mathbb{Z}^3) \end{cases}$$

for some $0 < \alpha(p), \beta(p) < \infty$.

-

Plaquette Percolation

- Construct a random complex by starting with the full integer lattice and adding 2-dimensional faces with probability p independently.

Theorem (Aizenman, Chayes, Chayes, Frölich, Russo)

Let γ be a planar rectangular loop in the integer lattice $\mathbb{L} \subset \mathbb{R}^3$, and let W_γ be the event that there is a plaquette surface with γ as its boundary. Then we have

$$\mathbb{P}(W_\gamma) \sim \begin{cases} \exp(-\alpha(p)\mathrm{Area}(\gamma)) & p < 1 - p_c(\mathbb{Z}^3) \\ \exp(-\beta(p)\mathrm{Per}(\gamma)) & p > 1 - p_c(\mathbb{Z}^3) \end{cases}$$

for some $0 < \alpha(p), \beta(p) < \infty$.

-